Sefton Park Schools

Mathematics:
Policy on teaching pencil and paper procedures for calculations

Copy incorporating National Curriculum and including update on division
July 2015

Progression towards standard written methods of calculation.

INTRODUCTION

Pencil and paper procedures will be taught as part of a structured and systematic approach to teaching number. We regard it as crucial that pupils develop good ‘number sense’ and understand the mathematics which underpin these procedures, enabling them to recall and apply a wide range of mental strategies to calculate efficiently and accurately. A considerable emphasis on teaching mental calculation strategies and informal written recording will be retained alongside the teaching of pencil and paper procedures,

REASONS FOR USING WRITTEN METHODS

- To aid calculation when the problem is too complex to be done mentally
- To aid mental calculation by writing down some of the numbers and answers involved
- To make clear a mental procedure for the pupil
- To help communicate methods and solutions
- To develop and refine a set of rules for calculations

We have developed a consistent approach to the teaching of written calculation methods. This will establish continuity and progression throughout the school.

MENTAL METHODS will be established first, BASED ON A GOOD UNDERSTANDING OF PLACE VALUE IN NUMBERS

The concept of place value will be introduced in Early Years taught from Year 1. Bead bars, bead frames, numicon and dienes will be used as the core teaching resources, progressing to number lines (to 10, 20 then 100) in Years 1 and 2.

The empty number line will then be introduced to aid calculations (

General guidance

- When teaching pencil and paper procedures examples that justify the procedure will be used as far as possible.
- Pupils will be encouraged to estimate first, then calculate, then check.
- Pupils will be taught to look out for special cases that can still be done entirely mentally.
- Pupils will need to develop a firm understanding of the underlying processes before progressing from expanded methods to compact layouts. Teachers will use their assessments to gauge when this is appropriate for classes and individuals, using the age related guidance in the National Curriculum Programmes of Study and this policy.
- Teaching of new procedures should involve opportunities for children to try the method alongside teacher modelling, before moving towards using it with increasing independence.

Guidance to support teachers in making judgement as to when children have been well prepared for written procedures:

Addition and subtraction

- Do they know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers in a variety of ways?
- Can they add three single digit numbers mentally?
- Can they add and subtract any pair of two digit numbers mentally?
- Do they understand zero as a place holder?
- Can they explain their mental strategies orally and record them using informal jottings?

Multiplication and division

- Do they know the 2, 3, 4, 5 and 10 time table
- Do they know the result of multiplying by 0 and 1?
- Do they understand 0 as a place holder?
- Can they multiply two and three digit numbers by 10 and 100?
- Can they double and halve two digit numbers mentally?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?

The above lists are not exhaustive but are a guide for the teacher to judge when a child is ready to move from informal to formal methods of calculation.

As a school we recognise the possibility that children will give greater ‘status’ or feel more comfortable with one or other method. We will present a variety of calculations (those which are available mentally, those which require jottings to ensure accuracy, and those which are most efficiently solved by using a written method) and involve children in making decisions about the most appropriate method for each calculation.

Addition and subtraction

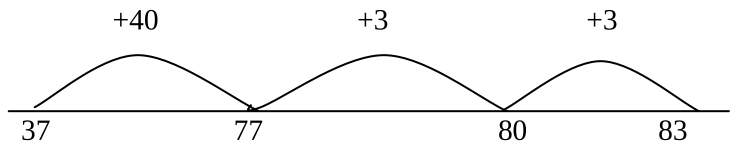
Year 2

Time will be used to **develop children's mental calculation skills** with adding and subtracting numbers using concrete objects, pictorial representations and mentally including:

- A two digit number and ones
- A two digit number and tens
- Two two-digit numbers
- Adding three one digit numbers

e.g. when calculating 37 + 46,
having made extensive use of concrete equipment which has developed understanding of place value, children will be encouraged to move to the method in which the first number is not partitioned

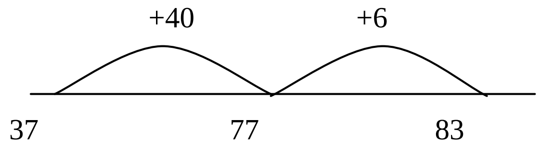
The empty number line will be used to model this;



The aim is that the recording is reduced to 'jottings'

e.g. 37 + 46 = Jottings 77 6

and;



Finally it is not needed at all. The calculation is done entirely mentally.

Addition and subtraction overview KS2

Year 3

Time will continue to be used to **develop children's mental calculation skills** with adding and subtracting mentally

- A three digit number and ones
- A three digit number and tens
- A three digit number and hundreds

and the empty number line used to support breaking numbers down,

Leading up to children being taught to add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.
Pupils will need to be able to calculate examples which bridge tens and/or hundreds. Examples where tens and/or hundreds are not bridged should not be used extensively, as these would not justify the procedure.

e.g 625 + 42 - This is easily 'available' mentally

Year 4

In Year 4 children will be taught to add and subtract numbers with up to 4 digits using formal written methods of columnar addition **where appropriate.** Time will continue to be spent on developing children's mental calculation skills. Calculations will be presented which require children to decide on the most appropriate strategy for each, considering which are available mentally or with jottings and which are most efficiently solved with a formal column method.

Year 5

In year 5 children will be taught to add and subtract numbers with more than 4 digits, including using formal written column methods. They will continue to retain a focus on adding and subtracting mentally with increasingly large numbers. They will be deciding on appropriate strategies and operations when solving two step problems involving addition and subtraction.

Year 6

In year 6 children should be making decision and applying efficient methods when solving multi-step problems involving addition and subtraction.

Progression in addition strategies

Adding the least significant digits first will be introduced as a preparation for the formal columnar method. Vertical recording of steps will be carried out alongside the same calculation recorded on a numberline.

76+47

76
+ 7
83
+ 40
123

Before moving to:

358
+ 73
11
120
300
431 — The numbers are added mentally from the bottom

When children are confident with this they can move onto the more compact layout. **As teachers talk through the compact layout the language should retain the place value of the digits**

Eg. ‘ 625
+ 48
673
1

‘Five and eight are thirteen. Put down the three and carry the ten. Twenty and forty are sixty and another ten is seventy. Put that down in the tens place. And now, another six hundred.

When the standard written method is introduced with an increasing number of digits we will model the expanded layout then work through the compact layout to show the link. The language of place value will be retained and kept consistent.

587 + 475

```
   587                        587
+  475      ———————→       + 475
    12                      1062
   150                       11
   900
  1062
```

Extend to adding several numbers with a different numbers of digits and to decimals

Gradually the children's language will compact but the teacher should continue to retain the focus on the place value of the digits in talking through the procedures.

Progression in subtraction strategies

As for addition, the emphasis should be on **developing mental methods.** The number line will be used for this before moving to the vertical layout. Children will be introduced to vertical recording of subtraction calculations alongside the same calculation carried out using a number line. Dienes will also be used to model what is happening when exchanges are being carried out.

N.B. It is important that children are secure with 'finding the difference' as well as the 'taking away' model of subtraction. Children will need to be given lots of practice in identifying when each of these models, as well as the formal written method is appropriate.

Work will be covered on **'Further partitioning' as this is a prerequisite skill for decomposition.**

Eg. **46 = 40 + 6**
46 = 30 + 16
46 = 20 + 26 etc.

As both complementary addition and decomposition with be taught, it is important that children are taught to select the best method for a calculation based on looking at the numbers involved.

The 'partitioning both numbers' strategy for examples such as 86 – 42 will not be used, as children readily and mistakenly transfer the method to deal with examples such as 82- 46.This will be made explicit as a possible misconception.

So, for example:
8003-4998 should be available mentally by upper key stage 2;
8006-4784
is more efficiently done by complementary addition as when the first number has several zeros, decomposition is less efficient; however

8233-4787

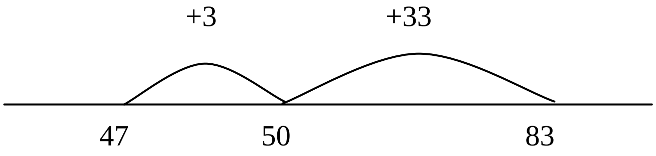
is more efficiently done using decomposition.

Complementary addition

The number line will be used to model complementary addition. It is important that appropriate examples are used.

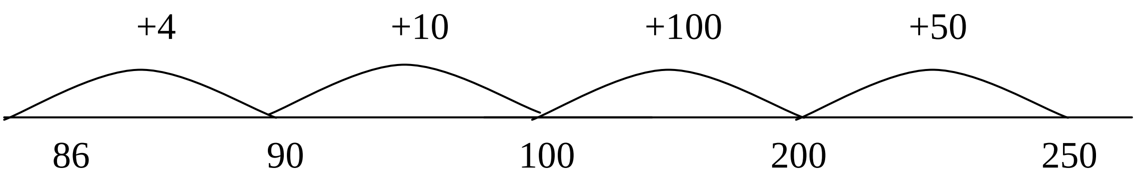
So 754 – 86 is appropriate whereas 786 – 54 is not. The latter example is available mentally.

83 - 47



Answer: 36

250-86



As children’s mental confidence increases they will be encouraged to make fewer jumps (NB children will be encouraged to draw more proportional jumps than in this diagram)

Subtracting the least significant digits first will be introduced as a preparation for the formal columnar method. Vertical recording of steps will be carried out alongside the same calculation recorded on a numberline.

267-149

$$\begin{array}{r} 267 \\ -\ \ \ 9 \\ \hline 258 \\ -\ \ 40 \\ \hline 218 \\ -100 \\ \hline 118 \end{array}$$

Decomposition will be taught

This layout should be used

754 – 86

	600	140	1
	~~700~~	~~50~~	4
-		80	6
	600	60	8

The teaching ‘patter’ should be as follows.

‘Four subtract 6; that gives me some problems **in this layout** so I’ll partition 54 into 40 and 14. Now I’ll be able to subtract the 6 from the 14. Forty subtract eighty – that also gives me a problem **in this layout**; so I’ll partition 740 into 600 and 140. Now I can subtract 80 from 140.

754 – 286

	600	140	1
	~~700~~	~~50~~	4
-	200	80	6
	400	60	8

leading to

6 14 1
~~75~~4
286
468

When the children are secure in their understanding of how this method works, they will be shown how it can be recorded using the compact layout.

	6 14 1
	~~75~~4
-	86
	668

These two layouts need to be modelled with links between the two made explicit so that understanding remains secure.

In this example, there is an adjustment in both the tens and the hundreds place.

Work should start using examples where there is only an adjustment in the tens place

Eg. 784 – 57

Some children may notice that this is available mentally, in which case, explain that you are using an example like this to introduce them to a method and then quickly move them on to calculations which justify the procedure such as the first example.

Multiplication

Grid method will be taught, followed by short multiplication once children are secure with the place value involved in multiplying larger numbers. When short multiplication is introduced the same calculation will be carried out alongside using grid method to make the link.

July 2015: partitioning has currently been taken out of the procedures for multiplication but this will be kept under review)

Children will explicitly NOT be taught to add a zero when multiplying by ten, as this can lead to misconceptions with multiplying decimals. If they notice this it will be discussed as the effect of making a number ten times larger.

Grid Method

x	10	4	
6	60	24	= 84

346 x 9 is approximately 350 x 10 = 3500

x	300	40	6	
9	2700	360	54	= 3114

Language for e.g. 20 x 8 ‘I know 2 x 8 is 16, so I know that 20 x 8 is 160 (ten times bigger)

Short method of multiplication (multiplying by a single digit number)

346 x 9   is approximately 350 x 10 = 3500

```
     346
x      9
    3114
     4 5
```

The teaching 'patter' is as follows: six nines are fifty-four (or nine sixes are fifty-four), put down the four, carry the fifty. Nine forties are three hundred and sixty, and another fifty is four hundred and ten, put down the ten and carry the four hundred. Nine three hundreds are two thousand seven hundred, and another four hundred is three thousand one hundred.

Once children are secure in their understanding of the procedure they can be shown how to compact the language to increase efficiency.

Long Multiplication (multiplying by a two-digit number
: When being introduced this should be modelled alongside the same calculation **placed in the grid.**

72 x 38 is approximately 70 x 40 = 2800

```
              72
           x  38
72 x 8       576     note: the small 1 indicating 10 from 8x2=16
                1
72 x 30     2160
            2736
              1
```

When multiplying the 72 by 30 the following 'patter' is suggested.
'I'm going to multiply 72 by 30 so I'll put a zero here and then multiply by three. That will make my answer ten times bigger**. At this point children should understand multiplication and division by powers of ten and understand the effect**.

Notes on teaching division in Years 1 & 2

It is important that sufficient emphasis is given to teaching the grouping model of division in Y1 and Y2 so that children move into Y3 with a secure understanding of division as repeated subtraction. This links to the pencil and paper procedure that is introduced.
ITPs that enhance the teaching of simple division and division with remainders are;

- Grouping
- Remainders

It is also recommended that teachers use the following Key Questions

How many times can I take a group of (the divisor) away from (the dividend)?

Or more succinctly;

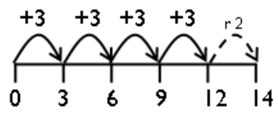
How many (the divisor) are there in (the dividend)?

Some exceptions are;

- When the divisor is 2. In this case it should trigger 'it's asking me to halve the number.'(sharing)
- When the divisor is a power of 10, children should understand how to shift the digits the appropriate number of places to the right.

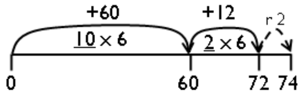
It is important that children are provided with opportunities to represent the repeated addition model of division using a numberline

14 ÷ 3 = 4 r 2



Chunking on a number line

Before children are introduced to the pencil and paper procedure outlined in the policy they need to be confident with partitioning two digit numbers to divide and representing this on numberlines
e.g.
74 ÷ 6 = 12 r 2



Chunking method for division (currently under review)
Informal written method
Using multiples of the divisor

e.g 78 ÷ 6

Key Question: ‘ How many times can I take a group of 6 away from 78?’

I know I can take at least 10 groups away because I know that 10 x 6 = 60

So,

```
   78
 - 60    (10 x 6)
   18                How many more groups of 6 can I take away?
 - 18    (3 x 6)     Three, because 3 x 6 = 18
    0
```

Answer: 13

Then use examples with remainders

78 ÷ 6

```
 6 | 78
 -  60      (10 x 6)
    18
 -  18      (3 x 6)
     0
```

Answer: 13

NB: It is important that appropriate examples are used for short division. 62 ÷ 6 is not appropriate because it is within table facts (simple division with remainders)

The progression is to start to use larger numbers in the dividend and to teach the children to subtract larger groups of the divisor by approximating first

e.g. 196 ÷ 6

```
  196
- 180   (30 x 6)
 ____
   16
-  12   (2 x 6 )
 ____
    4
```

Approximation:
I know 3 x 6 = 18
so I know 30 x 6 = 180
Therefore, I'll take 30 lots of 6 away.
40 lots of 6 would be too many because
4 x 6 = 24
so, 40 x 6 = 240
I've only got 196

This can be put into the layout below which looks like the standard layout although the method and language used is that described above.

```
     32   R4
6 | 196
  | 180
    16
    12

     4
```

Progressing to 'bus top method'

```
    1  5  9  r1
4 | 6 ²3 ³7
```

Which when introduced will be modelled alongside dienes to demonstrate the exchanging and place value which underpins this.

Short method may also be appropriate for some calculations involving two-digit divisors using the division gate e.g.

```
      27
36 | 977
   | 720
     257
     252
       5
```

Approximation

I know 2 x 36 = 72

so 20 x 36 = 720

I'll put a 2 in the tens column. There is now a need for 'side' calculations to get the nearest multiple of 36 to 257. Knowing that it was more than 5 times 36, 6 x 36 might be tried and then 7 x 36.

```
  36
x  7
 252
```

I'll put the 7 in the ones column

Answer: 27 R 5 or 27 $\frac{5}{36}$

547 ÷ 23 =

```
     2  3  r18
23 | 5  4  ⁸7
```

547 ÷ 23 = 23 r18

Long division

```
      2
15 | 3640
    -30
      6
```

```
     24
15 | 3640
    -30
      64
    - 60
       4
```

```
     242
15 | 3640
    - 30
      64
    - 60
       40
     - 30
       10
```